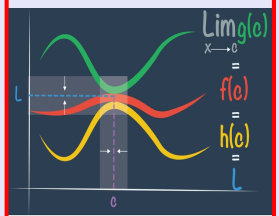
Math 261

Spring 2023

Lecture 17



Feb 19-8:47 AM

Given $f(x) = 3x^2 - 5x + 6$

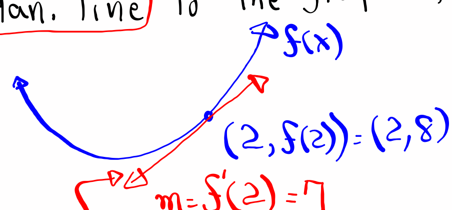
1) Find $f(2) = 3(2)^2 - 5(2) + 6 = 12 - 10 + 6 = \boxed{8}$

2) Find $f'(x) = \frac{d}{dx}\left[3x^2 - 5x + 6\right] = \frac{d}{dx}\left[3x^2\right] - \frac{d}{dx}\left[5x\right] + \frac{d}{dx}\left[6\right]$

$= 3 \cdot 2x - 5 \cdot 1 + 0 = \boxed{6x - 5}$

3) Find $f'(2) = 6(2) - 5 = \boxed{7}$

4) Find eqn of tan. line to the graph of $f(x)$ at $x = 2$.



$y - y_1 = m(x - x_1)$

$y - 8 = 7(x - 2)$

$y = 7x - 14 + 8 \rightarrow \boxed{y = 7x - 6}$

Mar 7-8:35 AM

Given $f(x) = \sqrt[3]{x}$

Recall $\sqrt[n]{x} = x^{\frac{1}{n}}$

So $\sqrt[3]{x} = x^{\frac{1}{3}}$

1) Find $f(8) = \sqrt[3]{8} = \boxed{2}$

2) Find $f'(x) = \frac{d}{dx}\left[\sqrt[3]{x}\right] = \frac{d}{dx}\left[x^{\frac{1}{3}}\right] = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-2/3}$

$= \frac{1}{3x^{2/3}} = \boxed{\frac{1}{3\sqrt[3]{x^2}}}$

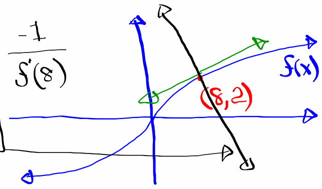
3) Find $f'(8)$

$= \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3\sqrt[3]{64}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

4) Find eqn of the normal line to the graph of $f(x)$ at $x=8$.

$m_{\text{Normal line}} = \frac{-1}{m_{\text{tan. line}}} = \frac{-1}{f'(8)}$

$= \frac{-1}{\frac{1}{12}} = \boxed{-12}$



$y - y_1 = m(x - x_1)$

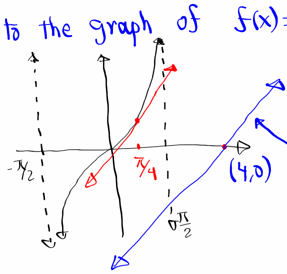
$y - 2 = -12(x - 8) \Rightarrow y = -12x + 96 + 2$

$\boxed{y = -12x + 98}$

Mar 7-8:56 AM

Find eqn of a line with x-Int at $x=4$ and is parallel to the tan. line at $x = \frac{\pi}{4}$ to the graph of $f(x) = \tan x$.

Parallel lines have Same slope



$m = m_{\text{tan. line}}$ at $x = \frac{\pi}{4}$

$f(x) = \tan x$

$f'(x) = \frac{d}{dx}\left[\tan x\right]$

$= \sec^2 x$

$m_{\text{tan. line}} = f'\left(\frac{\pi}{4}\right)$

$= \sec^2 \frac{\pi}{4}$

$= \left(\sqrt{2}\right)^2$

$= 2$

$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}}$

$= \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}}$

$= \frac{2\sqrt{2}}{2} = \sqrt{2}$

Now eqn of the line with x-Int at $x=4$

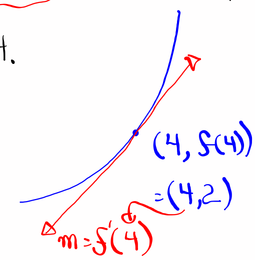
$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 4) \Rightarrow \boxed{y = 2x - 8}$

Mar 7-9:06 AM

Find eqn of the tan. line to the graph of

$$f(x) = \frac{x}{x-2} \quad \text{at } x = 4.$$



$$f(4) = \frac{4}{4-2} = \frac{4}{2} = 2$$

$$f'(x) = \frac{d}{dx}\left[\frac{x}{x-2}\right] = \frac{\frac{dx}{dx}\cdot(x-2) - x\cdot\frac{d}{dx}[x-2]}{(x-2)^2}$$

Recall from yesterday

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

$$= \frac{1(x-2) - x\cdot 1}{(x-2)^2}$$

$$= \frac{-2}{(x-2)^2}$$

$$m_{\text{tan. line}} = f'(4) = \frac{-2}{(4-2)^2} = \frac{-2}{4} = \boxed{\frac{-1}{2}}$$

$$y - y_1 = m(x - x_1) \qquad y - 2 = \frac{-1}{2}(x-4)$$

$$y = \frac{-1}{2}x + 2 + 2 \Rightarrow \boxed{y = \frac{-1}{2}x + 4}$$

Mar 7-9:15 AM

Prove $\displaystyle \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$

$$\frac{\frac{1}{2} - \frac{1}{3}}{5} \qquad \text{LCD} = 2\cdot 3$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

LCD $= g(x+h)\cdot g(x)$

$$= \lim_{h\to 0} \frac{\cancel{g(x+h)}\, g(x)\cdot \frac{f(x+h)}{\cancel{g(x+h)}} - g(x+h)\,\cancel{g(x)}\cdot\frac{f(x)}{\cancel{g(x)}}}{h\; g(x+h)\; g(x)}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x) - g(x+h)\cdot f(x)}{h\; g(x+h)\; g(x)}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x) - f(x)\cdot g(x) + f(x)\cdot g(x) - f(x)\cdot g(x+h)}{h\; g(x+h)\; g(x)}$$

$$= \lim_{h\to 0} \frac{g(x)\left[f(x+h) - f(x)\right] - f(x)\left[g(x+h) - g(x)\right]}{h\; g(x+h)\; g(x)}$$

Mar 7-9:28 AM

$$= \lim_{h \to 0} \frac{g(x)[f(x+h) - f(x)] - f(x)[g(x+h) - g(x)]}{h\, g(x+h)\, g(x)}$$

$$= \lim_{h \to 0} \frac{g(x)[f(x+h) - f(x)]}{h\, \cancel{g(x+h)}^{g(x)}\, g(x)} - \lim_{h \to 0} \frac{f(x)[g(x+h) - g(x)]}{h\, \cancel{g(x+h)}^{g(x)}\, g(x)}$$

$$= \frac{g(x)}{[g(x)]^2} \underbrace{\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}}_{f'(x)} - \frac{f(x)}{[g(x)]^2} \underbrace{\boxed{\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}}}_{g'(x)}$$

$$= \frac{g(x)\ f'(x)}{[g(x)]^2} - \frac{f(x) \cdot g'(x)}{[g(x)]^2}$$

$$= \frac{f'(x)\, g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

Mar 7-9:37 AM

Find $\frac{d}{dx}[\cot x]$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\frac{d}{dx}[\cos x] \cdot \sin x - \cos x \cdot \frac{d}{dx}[\sin x]}{[\sin x]^2}$$

$$= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= \boxed{-\csc^2 x}$$

Recall

$\csc x = \frac{1}{\sin x}$

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

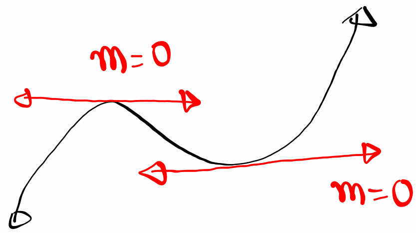
$\frac{d}{dx}[\sec x]$

$\frac{d}{dx}[\csc x]$

Mar 7-9:44 AM

Find all points on the graph of
$f(x) = \frac{1}{3}x^3 - 4x$ with horizontal tan. line.

$f'(x) = \frac{1}{3} \cdot 3x^2 - 4$



$f'(x) = x^2 - 4$

$f'(x) = 0$ $\quad x^2 - 4 = 0 \quad x = \pm 2$

Make sure to finish this.

$(-2, f(-2)) = (-2, \quad)$

$(2, f(2)) = (2, \quad)$